cube roots of unity. Then the roots are $m+n$, $\omega m+\omega^2 n$, $\omega^2 m+\omega n$. When $b^2/4=a^3/27$, or $a^3/b^2=\frac{27}{4}=6.75$, $m=n=\pm\frac{1}{2}\sqrt[3]{(4b)}$.

$\therefore m+n=\pm\sqrt[3]{(4b)}$, $\omega m+\omega^2 m=\mp\frac{1}{2}\sqrt[3]{(4b)}$, $\omega^2 m+\omega n=\mp\frac{1}{2}\sqrt[3]{(4b.)}$

When $b^2/4<a^3/27$, or $a^2/b^2<\frac{27}{4},=6.75$, $\sqrt{(b^2/4-a^3/27)}$ is imaginary.

Let $m=u+\sqrt{(-1)}v$, $n=u-\sqrt{(-1)}v$.

$\therefore m+n=u+v$, $\omega m+\omega^2 n=-u-v\sqrt{3}$, $\omega^2 m+\omega n=-u+v\sqrt{3}$, all real and unequal.

When $a^3/b^2>\frac{27}{4},=6.75$, $\sqrt{\dfrac{b^2}{4}-\dfrac{a^3}{27}}$ is real.

$\therefore m+n$ is real, and $\omega m+\omega^2 n$, $\omega^2 m+\omega n$ are imaginary.

J. W. Clawson, of Collegeville, Pa., referred to Burnside and Panton's *Theory of Equations*, Vol. I, §§42, 43. Discussions of this problem are to be found in nearly all texts on the Theory of Equations.

296. Proposed by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philahelphia, Pa.

Sum the series, $1+\frac{1}{6}+\frac{1}{20}+\frac{1}{50}+\frac{1}{105}+\frac{1}{196}+\frac{1}{336}+\ldots$

Solution by A. R. MAXSON, A. M., Columbia University, New York.

In the series 1, 6, 20, 50, 105, 196, 336, ..., the successive orders of differences are,

$$\begin{array}{c} 5,\ 14,\ 30,\ 55,\ 91,\ 140,\ \ldots \\ 9,\ 16,\ 25,\ 36,\ 49,\ \ldots \\ 7,\ 9,\ 11,\ 13,\ \ldots \\ 2,\ 2,\ 2,\ \ldots \\ 0,\ 0,\ \ldots \end{array}$$

The nth term is then

$$1+5(n-1)+\frac{9}{2\,!}(n-1)(n-2)+\frac{7}{3\,!}(n-1)(n-2)(n-3)$$

$$+\frac{2}{4\,!}(n-1)(n-2)(n-3)(n-4)=\frac{n}{12}(n+2)(n+1)^2.$$

The nth term of the given series is then $\dfrac{12}{n(n+2)(n+1)^2}$, which can be written $\left(\dfrac{6}{n}+\dfrac{6}{n+1}\right)-\left(\dfrac{6}{n+1}-\dfrac{6}{n+2}\right)-\dfrac{12}{(n+1)^2}$.

Taking now u_r as the rth term of the original series, we have

$$u_1=\left(\frac{6}{1}+\frac{6}{2}\right)-\left(\frac{6}{2}+\frac{6}{3}\right)-12.\frac{1}{2^2},$$

$$u_2=\left(\frac{6}{2}+\frac{6}{3}\right)-\left(\frac{6}{3}+\frac{6}{4}\right)-12.\frac{1}{3^2},$$

$$\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot\quad\cdot$$

$$u_{n-1}=\left(\frac{6}{n-1}+\frac{6}{n}\right)-\left(\frac{6}{n}+\frac{6}{n+1}\right)-12.\frac{1}{n^2}.$$

By addition, we have $\sum\limits_{r=1}^{r=n-1} u_r=9-\frac{6}{n}-\frac{6}{n+1}+12-12\Sigma\frac{1}{n^2}$. For the sum to infinity we have $\sum\limits_{r=1}^{r=\infty} u_r=21-12.\frac{\pi^2}{6}=21-2\pi^2$, on remembering that $\sum\limits_1^\infty\frac{1}{n^2}=\frac{\pi^2}{6}$.

Also solved by G. B. M. Zerr, J. W. Clawson, and H. V. Spunar.

297. Proposed by W. J. GREENSTREET, Marling School, Stroud, England.

If a, b, c, d, f, g, h are all real, and a, $ab-h^2$, $abc+2fgh-af^2-bg^2-ch^2$ are all positive, show that b, c, $bc-f^2$, and $ca-g^2$ are also positive.

I. Solution by C. R. MacINNIS, Princeton, N. J.

Since both a and $ab-h^2$ are positive, b must be positive.

$$abc+2fgh-af^2-bg^2-ch^2\equiv\frac{(ab-h^2)(bc-f^2)-(hf-bg)^2}{b}.$$

Since the whole expression is positive and both b and $ab-h^2$ are also positive, $bc-f^2>0$. Hence $c>0$. Similarly,

$$abc+2fgh-af^2-bg^2-ch^2\equiv\frac{(ab-h^2)(ca-g^2)-(hg-af)^2}{a},\text{ and } ca-g^2>0.$$

II. Solution by A. F. CARPENTER, Hastings, Nebr.

Since $ab-h^2$ is positive $ab>h^2$, and since h is real, h^2 is positive. Then ab, which is greater than h^2, is positive. But a is positive; hence b is positive.

Now $b(abc+2fgh-af^2-bg^2-ch^2)=(ab-h^2)(bc-f^2)-(bg-fh)^2$; that is, $(bc-f^2)(ab-h^2)=b$ (a positive quantity)$+(bg-fh)^2=a$ positive quantity, and since $ab-h^2$ is positive, $(bc-f^2)$ is positive.

Again, $a(abc+2fgh-af^2-bg^2-ch^2)=(ab-h^2)(ca-g^2)-(af-hg)^2$, and it follows as before that $(ca-g^2)$ is positive.